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417A. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Solve $E(x^2) - E(3x) = 7$, where $E(m)$ is the largest integer in m. The value of x is to be in the form $4 + (y/32)$ where y is an integer.

This problem was incorrectly numbered 417.

SOLUTION BY A. M. HARDING, University of Arkansas.

If we draw the graphs of the functions $E(x^2)$ and $E(3x) + 7$ we find

$$E(x^2) = E(3x) + 7 = 20, \quad \text{if} \quad \sqrt{20} < x < \sqrt{21}; \quad \text{i. e.} \quad 4.4721 < x < 4.5826;$$

$$E(x^2) = E(3x) + 7 = 21, \quad \text{if} \quad 14/3 < x < \sqrt{22}; \quad \text{i. e.} \quad 4.6667 < x < 4.6904;$$

$$E(x^2) = E(3x) + 7 = 2, \quad \text{if} \quad -5/3 < x < -\sqrt{2},$$

$$\text{i. e.} \quad -1.6667 < x < -1.4142.$$

Now we must have $x = 4 + y/32$ where y is an integer. Hence

$$4.4721 < 4 + y/32 < 4.5826, \qquad \therefore y = 16, 17, \text{ or } 18.$$

And

$$4.6667 < 4 + y/32 < 4.6904, \qquad \therefore y = 22.$$

$$-1.6667 < 4 + y/32 < -1.4142, \qquad \therefore y = -181, -180, \cdots, -174.$$

GEOMETRY.

430. Proposed by DANIEL KRETH, Wellman, Iowa.

The distance between A and B is always a feet. A travels along a straight path at the rate of v_1 miles per hour, and B starts at the same time in the path behind A and travels in a curve at the rate of v_2 miles per hour. How far will B travel to reach the path in front of A, and how far to reach the path again behind A?

SOLUTION BY ELIJAH SWIFT, University of Vermont.

Take the origin at the place where B starts and the Y-axis through A. Then the coördinates of A at any time, t, are $(0, a + v_1t)$, and those of B must satisfy the equations

$$(1) \quad x^2 + (y - a - v_1t)^2 = a,^2 \qquad (2) \quad \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = v_2^2.$$

Solving (1) for y, we have $y = -\sqrt{a^2 - x^2} + a + v_1t$, where the negative sign holds until B is abreast of A.

Substituting this value in (2), we have a quadratic equation in dx/dt, from which we find

$$\frac{dx}{dt} = \frac{\sqrt{a^2 - x^2}}{a^2}\{-v_1x + \sqrt{v_1^2x^2 - a^2v_1^2 + a^2v_2^2}\},$$

the plus sign of the second radical being taken since dx/dt is positive.

From equation (2), we deduce $ds = v_2dt$. Hence, we have

$$s \equiv v_2t = 2 \cdot v_2 \cdot a^2 \int_0^a \frac{dx}{\sqrt{a^2 - x^2}\{-v_1x + \sqrt{v_1^2x^2 - a^2v_1^2 + a^2v_2^2}\}},$$

as the total distance is evidently twice as far as the distance B must cover to be abreast of A. This would seem to be an elliptic integral and hence not easily evaluated.

The same reasoning would hold on the return, except for signs, and we obtain the same expression for s except for the sign of v_1x, which would be plus.

Note.—This problem should evidently have been listed under Calculus.

432. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Having given a tetrahedron, a, b, c, d, e, f, find an expression for the radius of the sphere which is tangent to the six edges.

SOLUTION BY J. W. CLAWSON, Collegeville, Pa.

First, a solution is not possible in the general case. Eight spheres can be drawn to touch any four of the six edges, but none of them will touch the other two edges unless certain conditions are satisfied.

If the tetrahedron is of such a shape that a sphere can be *inscribed* to touch all the edges, the planes of the four faces of the tetrahedron cut the sphere in circles inscribed to the triangles which form the faces. At each point where one of these four circles touches an edge, another of the circles also touches it. Now if T is the point where the in-circle of the triangle ABC, whose edges are a, b, c touches the side AC, then $CT = (a + b - c)/2$; and if T is the point where the in-circle of the triangle ACD whose edges are f, d, b touches the side AC, then $CT = (b + f - d)/2$.

Therefore $a + d = c + f$.

Similarly it can be shown that $a + d = b + e$.

Hence, a sphere can be drawn to touch the six edges internally if

$$a + d = b + e = c + f \tag{1}$$

Similarly it can be shown that a sphere can be drawn tangent to the six edges, touching b, c, d *produced*, if $a - d = e - b = f - c$; that a sphere can be drawn tangent to the six edges, touching a, b, c *produced* if $a - d = e - b = c - f$; one touching a, b, f, *produced* if $a - d = b - e = f - c$; one touching e, d, f *produced* if $a - d = b - e = c - f$.

Thus, if either

$$a + d = b + e = c + f,$$

$$\text{or}\quad a - d = b - e = c - f, \qquad \text{or}\quad a - d = b - e = f - c,$$

$$\text{or}\quad a - d = e - b = c - f, \qquad \text{or}\quad a - d = e - b = f - c,$$

a sphere can be found to touch the six edges.

If any face is an equilateral triangle and the other three edges are equal, two such spheres can be drawn, one inscribed, the other escribed, touching the sides of the equilateral triangle and the three other edges produced.